MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2022/2023 (TERM ID 2210)

PES0024 – ESSENTIAL STATISTICS

(All sections / Groups)

10 FEBRUARY 2023
9.00 a.m. - 11.00 a.m.
(2 hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of 6 pages **excluding** the cover page.
2. This question paper consists of **FOUR** structured questions. Attempt **ALL** questions.
3. Students are allowed to use non-programmable scientific calculators with no restrictions.
4. The **formulae** and **z-tables** are attached at the end of the question paper.
5. **Statistical book** is provided.

QUESTION 1 [25 Marks]

a) A restaurant chain routinely surveys its customers whether he or she would return to the restaurant or not and to rate the quality of food. From 300 customers, it recorded the following:

Rating on Food Quality	Will Return	Will not return
Poor	6	30
Good	129	69
Excellent	60	6

(i) What is the probability that the customers rate the food as good? [3 marks]
(ii) What is the probability that the customer rate the food as excellent and will return to the restaurant? [2 marks]

b) There are 25 students in a class. Five of them scored A and 10 of them scored B while the others scored C for Calculus. If a student is selected at random, what is the probability that the selected student scored A or B in Calculus? [3 marks]

c) Given the following probability density function,

$$f(x) = \begin{cases} k(x^2+1) & 1 \le x \le 4 \\ 0 & elsewhere \end{cases}$$

(i) Find the value of constant, k. [4 marks]
(ii) Calculate $P(2 < X \le 3)$. [3 marks]
(iii) Find the cumulative distribution function, $F(x)$. [6 marks]
(iv) Find the expected value of the random variable X. [4 marks]

QUESTION 2 [30 Marks]

a) According to a public census for a particular city, one-fourth of all businesses are owned by women. If we randomly selected 17 businesses,

(i) What is the probability that from 5 to 10 of them are owned by women? [4 marks]
(ii) What is the probability at least 8 businesses are owned by women? [2 marks]
(iii) What is the probability not more than 6 businesses are owned by women? [2 marks]
(iv) Calculate the expected value and standard deviation for the number of women who owned a business. [4 marks]

Continued...

b) Automobiles arrive at the Sunway Toll exit at the rate of two per minute. The distribution of arrivals approximates a Poisson distribution.

(i) What is the probability that no automobiles arrive in a particular minute? [3 marks]

(ii) What is the probability that at least one automobile arrives during a particular minute? [3 marks]

(iii) What is the average number of automobile arrivals during half an hour? [2 marks]

c) A recent national survey concluded that the typical person consumes 48 ounces of water per day. Assume daily water consumption follows a normal probability distribution with a standard deviation of 2.8 ounces. Find the probability that

(i) a person consumes more than 52 ounces of water per day. [5 marks]

(ii) a person consumes between 40 to 50 ounces of water per day. [5 marks]

QUESTION 3 [25 Marks]

a) The following data represent the quantities ordered (in million units) for the first five weeks for a certain product in Deen company:

14.5 19.0 11.0 15.5 24.0

(i) Assuming that you sample without replacement, select all possible samples of n=3 and compute the mean of all the samples. [7 marks]

(ii) Construct the sampling distribution of the samples. [5 marks]

(iii) Calculate the sampling error. [5 marks]

(iv) The order management officer selected a sample of 3 numbers; 19.0,11.0 and 24.0 from this population. Unfortunately, he recorded 2.0 instead of 24.0 for this sample, what is the non-sampling error occurred? [3 marks]

b) The owner of Britten's Egg Farm wants to estimate the mean number of eggs produced per chicken. The egg production is normally distributed with a mean of 20 eggs and a standard deviation of 2 eggs. If he selects a sample of 8 chickens, find the probability distribution that the sample mean is between 18 and 21 eggs. [5 marks]

Continued...

QUESTION 4 [20 Marks]

A researcher interested to study on the average rate of income tax paid among senior executives in private sector. A random sample of 15 senior executives of several firms is chosen and the amount each executive paid for income taxes as a percentage of gross income is recorded. Given that the population standard deviation is 5.0.

16.0	28.0	18.6	20.2	30.4	22.4	22.4	33.7
23.2	24.1	24.3	24.7	25.2	25.9	26.3	

a) Compute the point estimate of the population mean. [3 marks]

b) Compute the 95% margin of error. [3 marks]

c) Construct a 97% confidence interval of the mean rate of income tax paid. Interpret the interval. [8 marks]

d) Construct a 90% confidence interval of the mean rate of income tax paid given that the population standard deviation is unknown, and sample standard deviation was 4.42. [6 marks]

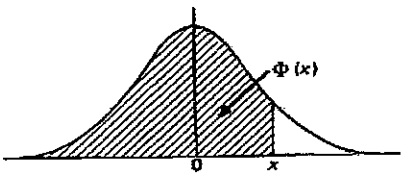
End of Question.

FORMULAE LIST

Discrete Random Variable	$E(X)=\sum x\cdot p(x)$ $E\left(X^2\right)=\sum x^2\cdot p(x)$ $V(X)=E\left(X^2\right)-[E(X)]^2$
Binomial	$P(X=x)=B(n,p)=\binom{n}{x}p^x q^{n-x}$ $\mu=np$ $\sigma=\sqrt{npq}$
Poisson	$P(X=x)=Poi(x,\lambda t)=\frac{(\lambda t)^x e^{-\lambda t}}{x!}$ $\mu=\lambda t$ $\sigma=\sqrt{\lambda t}$
Continuous Random Variable	$E(X)=\int_{-\infty}^{\infty}x\cdot f(x)dx$ $E\left(X^2\right)=\int_{-\infty}^{\infty}x^2\cdot f(x)dx$ $V(X)=E\left(X^2\right)-[E(X)]^2$
Standard normal	$z=\frac{x-\mu}{\sigma}$
Sample mean, $\bar{x}$	$z=\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$ or $z=\frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$
Confidence Interval for population mean	$\mu=\bar{x}\pm z\frac{\sigma}{\sqrt{n}}$ or $\mu=\bar{x}\pm z\frac{s}{\sqrt{n}}$

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

The function tabulated is $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2}\, dt$. $\Phi(x)$ is the probability that a random variable, normally distributed with zero mean and unit variance, will be less than or equal to x. When $x < 0$ use $\Phi(x) = 1 - \Phi(-x)$, as the normal distribution with zero mean and unit variance is symmetric about zero.



x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
0·00	0·5000	0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·97725
·01	·5040	·41	·6591	·81	·7910	·21	·8869	·61	·9463	·01	·97778
·02	·5080	·42	·6628	·82	·7939	·22	·8888	·62	·9474	·02	·97831
·03	·5120	·43	·6664	·83	·7967	·23	·8907	·63	·9484	·03	·97882
·04	·5160	·44	·6700	·84	·7995	·24	·8925	·64	·9495	·04	·97932
0·05	0·5199	0·45	0·6736	0·85	0·8023	1·25	0·8944	1·65	0·9505	2·05	0·97982
·06	·5239	·46	·6772	·86	·8051	·26	·8962	·66	·9515	·06	·98030
·07	·5279	·47	·6808	·87	·8078	·27	·8980	·67	·9525	·07	·98077
·08	·5319	·48	·6844	·88	·8106	·28	·8997	·68	·9535	·08	·98124
·09	·5359	·49	·6879	·89	·8133	·29	·9015	·69	·9545	·09	·98169
0·10	0·5398	0·50	0·6915	0·90	0·8159	1·30	0·9032	1·70	0·9554	2·10	0·98214
·11	·5438	·51	·6950	·91	·8186	·31	·9049	·71	·9564	·11	·98257
·12	·5478	·52	·6985	·92	·8212	·32	·9066	·72	·9573	·12	·98300
·13	·5517	·53	·7019	·93	·8238	·33	·9082	·73	·9582	·13	·98341
·14	·5557	·54	·7054	·94	·8264	·34	·9099	·74	·9591	·14	·98382
0·15	0·5596	0·55	0·7088	0·95	0·8289	1·35	0·9115	1·75	0·9599	2·15	0·98422
·16	·5636	·56	·7123	·96	·8315	·36	·9131	·76	·9608	·16	·98461
·17	·5675	·57	·7157	·97	·8340	·37	·9147	·77	·9616	·17	·98500
·18	·5714	·58	·7190	·98	·8365	·38	·9162	·78	·9625	·18	·98537
·19	·5753	·59	·7224	·99	·8389	·39	·9177	·79	·9633	·19	·98574
0·20	0·5793	0·60	0·7257	1·00	0·8413	1·40	0·9192	1·80	0·9641	2·20	0·98610
·21	·5832	·61	·7291	·01	·8438	·41	·9207	·81	·9649	·21	·98645
·22	·5871	·62	·7324	·02	·8461	·42	·9222	·82	·9656	·22	·98679
·23	·5910	·63	·7357	·03	·8485	·43	·9236	·83	·9664	·23	·98713
·24	·5948	·64	·7389	·04	·8508	·44	·9251	·84	·9671	·24	·98745
0·25	0·5987	0·65	0·7422	1·05	0·8531	1·45	0·9265	1·85	0·9678	2·25	0·98778
·26	·6026	·66	·7454	·06	·8554	·46	·9279	·86	·9686	·26	·98809
·27	·6064	·67	·7486	·07	·8577	·47	·9292	·87	·9693	·27	·98840
·28	·6103	·68	·7517	·08	·8599	·48	·9306	·88	·9699	·28	·98870
·29	·6141	·69	·7549	·09	·8621	·49	·9319	·89	·9706	·29	·98899
0·30	0·6179	0·70	0·7580	1·10	0·8643	1·50	0·9332	1·90	0·9713	2·30	0·98928
·31	·6217	·71	·7611	·11	·8665	·51	·9345	·91	·9719	·31	·98956
·32	·6255	·72	·7642	·12	·8686	·52	·9357	·92	·9726	·32	·98983
·33	·6293	·73	·7673	·13	·8708	·53	·9370	·93	·9732	·33	·99010
·34	·6331	·74	·7704	·14	·8729	·54	·9382	·94	·9738	·34	·99036
0·35	0·6368	0·75	0·7734	1·15	0·8749	1·55	0·9394	1·95	0·9744	2·35	0·99061
·36	·6406	·76	·7764	·16	·8770	·56	·9406	·96	·9750	·36	·99086
·37	·6443	·77	·7794	·17	·8790	·57	·9418	·97	·9756	·37	·99111
·38	·6480	·78	·7823	·18	·8810	·58	·9429	·98	·9761	·38	·99134
·39	·6517	·79	·7852	·19	·8830	·59	·9441	·99	·9767	·39	·99158
0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·9772	2·40	0·99180

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
2·40	0·99180	2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918
·41	·99202	·56	·99477	·71	·99664	·86	·99788	·01	·99869	·16	·99921
·42	·99224	·57	·99492	·72	·99674	·87	·99795	·02	·99874	·17	·99924
·43	·99245	·58	·99506	·73	·99683	·88	·99801	·03	·99878	·18	·99926
·44	·99266	·59	·99520	·74	·99693	·89	·99807	·04	·99882	·19	·99929
2·45	0·99286	2·60	0·99534	2·75	0·99702	2·90	0·99813	3·05	0·99886	3·20	0·99931
·46	·99305	·61	·99547	·76	·99711	·91	·99819	·06	·99889	·21	·99934
·47	·99324	·62	·99560	·77	·99720	·92	·99825	·07	·99893	·22	·99936
·48	·99343	·63	·99573	·78	·99728	·93	·99831	·08	·99896	·23	·99938
·49	·99361	·64	·99585	·79	·99736	·94	·99836	·09	·99900	·24	·99940
2·50	0·99379	2·65	0·99598	2·80	0·99744	2·95	0·99841	3·10	0·99903	3·25	0·99942
·51	·99396	·66	·99609	·81	·99752	·96	·99846	·11	·99906	·26	·99944
·52	·99413	·67	·99621	·82	·99760	·97	·99851	·12	·99910	·27	·99946
·53	·99430	·68	·99632	·83	·99767	·98	·99856	·13	·99913	·28	·99948
·54	·99446	·69	·99643	·84	·99774	·99	·99861	·14	·99916	·29	·99950
2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918	3·30	0·99952

The critical table below gives on the left the range of values of x for which $\Phi(x)$ takes the value on the right, correct to the last figure given; in critical cases, take the upper of the two values of $\Phi(x)$ indicated.

3·075	0·9990	3·263	0·9994	3·731	0·99990	3·916	0·99995
3·105	0·9991	3·320	0·9995	3·759	0·99991	3·976	0·99996
3·138	0·9992	3·389	0·9996	3·791	0·99992	4·055	0·99997
3·174	0·9993	3·480	0·9997	3·826	0·99993	4·173	0·99998
3·215	0·9994	3·615	0·9998	3·867	0·99994	4·417	0·99999
			0·9999		0·99995		1·00000

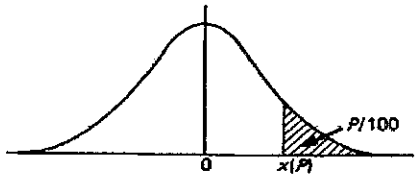
When $x > 3{\cdot}3$ the formula $1-\Phi(x) \doteqdot \frac{e^{-\frac{1}{2}x^2}}{x\sqrt{2\pi}}\left[1-\frac{1}{x^2}+\frac{3}{x^4}-\frac{15}{x^6}+\frac{105}{x^8}\right]$ is very accurate, with relative error less than $945/x^{10}$.

TABLE 5. PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

This table gives percentage points $x(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{2\pi}}\int_{x(P)}^{\infty} e^{-\frac{1}{2}t^2}\,dt.$$

If X is a variable, normally distributed with zero mean and unit variance, $P/100$ is the probability that $X \geqslant x(P)$. The lower P per cent points are given by symmetry as $-x(P)$, and the probability that $|X| \geqslant x(P)$ is $2P/100$.



P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$
50	0·0000	5·0	1·6449	3·0	1·8808	2·0	2·0537	1·0	2·3263	0·10	3·0902
45	0·1257	4·8	1·6646	2·9	1·8957	1·9	2·0749	0·9	2·3656	0·09	3·1214
40	0·2533	4·6	1·6849	2·8	1·9110	1·8	2·0969	0·8	2·4089	0·08	3·1559
35	0·3853	4·4	1·7060	2·7	1·9268	1·7	2·1201	0·7	2·4573	0·07	3·1947
30	0·5244	4·2	1·7279	2·6	1·9431	1·6	2·1444	0·6	2·5121	0·06	3·2389
25	0·6745	4·0	1·7507	2·5	1·9600	1·5	2·1701	0·5	2·5758	0·05	3·2905
20	0·8416	3·8	1·7744	2·4	1·9774	1·4	2·1973	0·4	2·6521	0·01	3·7190
15	1·0364	3·6	1·7991	2·3	1·9954	1·3	2·2262	0·3	2·7478	0·005	3·8906
10	1·2816	3·4	1·8250	2·2	2·0141	1·2	2·2571	0·2	2·8782	0·001	4·2649
5	1·6449	3·2	1·8522	2·1	2·0335	1·1	2·2904	0·1	3·0902	0·0005	4·4172

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